TECHNISCHE UNIVERSITÄT MÜNCHEN

Zentrum Mathematik

Dr. Stefan Possanner

Fallstudien der Mathematischen Modellbildung, part 3:
Asymptotic methods for perturbation problems, WS 2018/19

http://www-m16.ma.tum.de/Allgemeines/Fallstd18

Übungsblatt 3 (January 23th, 2019)

Regular perturbations of nonlinear IVPs

1. For $\varepsilon \in (0, \varepsilon_0]$, consider the followign IVP discussed in the lecture notes (Duffing equation),

$$\begin{aligned} &\frac{\mathrm{d}^2 x_\varepsilon}{\mathrm{d}t^2} + x_\varepsilon + \varepsilon x_\varepsilon^3 = 0\,, \qquad t > 0\,, \\ &x_\varepsilon(0) = 1\,, \qquad \frac{\mathrm{d}x_\varepsilon}{\mathrm{d}t}(0) = \varepsilon\,. \end{aligned} \tag{1}$$

   (a) Formulate this problem such that it fits the framework of Theorem 1 (see page 30 in the lecture notes) for approximating nonlinear IVPs: a) write it as a system of first-order ODEs for $u_\varepsilon : [0, \infty) \to \mathbb{R}^2$ and b) find the AEs of the vector field f and the initial condition V.

   (b) Make an ansatz for u_ε in the form of a regular AE and determine the IVPs satisfied by the coefficient functions u_0 and u_1.

   (c) Solve for u_0 and u_1 and state the domain of validity of the so obtained AE of x_ε.

2. Let u_ε be the solution of the IVP

$$\begin{aligned} &\frac{\mathrm{d}u_\varepsilon}{\mathrm{d}t} = 1 + u_\varepsilon^2 + \varepsilon u_\varepsilon\,, \qquad t > 0\,, \\ &u_\varepsilon(0) = \varepsilon\,. \end{aligned} \tag{2}$$

   We are intrested in approximations of u_ε when $\varepsilon \ll 1$.

   (a) Solve the reduced problem (Hint: consider trigonometric functions).

   (b) Compute an AE of u_ε with errors of $O(\varepsilon^2)$ and state its domain of validity.