TECHNISCHE UNIVERSITÄT MÜNCHEN
Zentrum Mathematik

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Fallstudien der Mathematischen Modellbildung, part 3:
Asymptotic methods for perturbation problems, WS 2018/19
`http://www-m16.ma.tum.de/Allgemeines/Fallstd18`
Übungsblatt 4 (January 30th, 2019)

Application of K.B.M. Theorem to weakly nonlinear oscillations

We consider weakly nonlinear oscillations as in the perturbation problem

$$P_\varepsilon \begin{cases} \dfrac{\mathrm{d}^2 x_\varepsilon}{\mathrm{d}t^2} + x_\varepsilon = \varepsilon f\left(x_\varepsilon, \dfrac{\mathrm{d}x_\varepsilon}{\mathrm{d}t}\right), & t \in \mathbb{R}, \\ x_\varepsilon(t_0) = \alpha, \qquad \dfrac{\mathrm{d}x_\varepsilon}{\mathrm{d}t}(t_0) = \beta, \end{cases} \tag{1}$$

where f is continuously differentiable in both arguments and $\alpha, \beta \in \mathbb{R}$ such that $\alpha^2 + \beta^2 \neq 0$.

1. Formulate the problem as a first-order IVP for the vector-valued unknown

$$u_\varepsilon := \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} x_\varepsilon \\ \frac{\mathrm{d}x_\varepsilon}{\mathrm{d}t} \end{pmatrix}.$$

2. Bring the problem for u_ε into standard form by computing the flow map $\Phi_0 : \mathbb{R} \times \mathbb{R}^2 \to \mathbb{R}^2$ of the reduced system.

3. Apply the K.B.M. theorem to derive an asymptotic approximation of u_ε valid for $t_0 \leq t \leq t_0 + \frac{t_1}{\varepsilon}$, where t_1 is independent of ε.

4. Apply your result from point 3. to the Duffing equation:

$$\frac{\mathrm{d}^2 x_\varepsilon}{\mathrm{d}t^2} + x_\varepsilon = -\varepsilon x_\varepsilon^3 . \tag{2}$$

   Compare to previous results for the Duffing equation obtained by the method of the strained coordinate.

5. Apply your result from point 3. to the Van Der Pol equation:

$$\frac{\mathrm{d}^2 x_\varepsilon}{\mathrm{d}t^2} + x_\varepsilon = \varepsilon(1 - x_\varepsilon^2)\frac{\mathrm{d}x_\varepsilon}{\mathrm{d}t} . \tag{3}$$